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MBA-003-1014008

Seat No. ______

B. Sc. (Sem. IV) (CBCS) Examination

March / April - 2018

MATH-04(A) : Linear Algebra & Differential Geometry

(Elective) (New Course)

Faculty Code : 003
Subject Code : 1014008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate full marks of the question.

1 (a) Answer the following : **4**

(1) If W_1 and W_2 are subspaces of a vector space V, then $W_1 \cup W_2$ subspace of V.

(True / False)

(2) Define : Linear span of a set.

(3) State whether the subset $\{(0, 0, 0), (1, 2, 3), (2, 1, 2)\}$ of R^3 is linearly independent or dependent.

(4) Define : Basic of a vector space.

(b) Attempt any **one** : **2**

(1) Prove that any superset of linearly dependent set is also linearly dependent.

(2) Let V be a vector space over $\mathbb{R}$. Prove that a non-empty subset W of a vector space V is a subspace of V if for all $c_1, c_2 \in \mathbb{R}$ and $w_1, w_2 \in W$, $c_1 w_1 + c_2 w_2 \in W$.

(c) Attempt any **one** : **3**

(1) Examine whether the set $W = \{(a, b, c) | a^2 + b^2 + c^2 \leq 1\}$ is a subspace of $\mathbb{R}^3$ or not.

(2) Prove that $(1, 0, -1) \in span(A)$ and $(0, -1, 1) \notin span(A)$, where $A = \{(2, 1, 0), (-1, 0, 1), (0, 1, 2)\}$.

(d) Attempt any **one** : **5**

(1) Verify whether the set $V = \{(a, b) | a, b \in \mathbb{R}, b > 0\}$ under the operations $(a, b) + (c, d) = (a + c, bd)$, $\alpha(a, b) = (a\alpha, b^{\alpha}), (a, b), (c, d) \in V, \alpha \in \mathbb{R}$ is a vector space over $\mathbb{R}$ or not ?

(2) Let V be a vector space and $A = \{v_1, v_2, ..., v_n\} \subset V$ be such that span $(A) = V$. If $w_1, w_2, ..., w_m$ are linearly independent vectors of a V then prove that $m \leq n$.

2 (a) Answer the following : **4**

(1) Define : Dimension of a vector space.

(2) What is the standard basis of the vector space $P_2(\mathbb{R})$?

(3) If W is a subspace of a finite dimensional vector space V, then dim W _____ dim V.

(4) If W is a subspace of a vector space V, then dim $\left(W \oplus W^{\perp}\right) =$ _____.

(b) Attempt any one : **2**

(1) Examine whether the subset

$A = \left\{\sin x, \cos x, \sin\left(x + \frac{\pi}{6}\right)\right\}$ of real vector space

$C[0, 2\pi]$ is linearly dependent or linearly independent.

(2) If W is a subspace of a finite dimensional vector space V such that dim W = dim V then prove that $W = V$.

(c) Attempt any one : **3**

(1) Prove that the intersection of two subspaces of a vector space V is also a subspace of V.

(2) Extend the subset $\left\{1 - x + x^2, 2x - x^2 + x^3\right\}$ of $P_3(\mathbb{R})$ to form a basis of $P_3(\mathbb{R})$.

(d) Attempt any one : **5**

(1) Show that the set $A = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of $\mathbb{R}^3$. Find coordinates of the vector (2, 1, –1) of $\mathbb{R}^3$ with respect to this basis.

(2) Let $W_1 = \{(x - y, y + z, y, z)\}$ and

$W_2 = \{(x, x + y, x + y + z, y - z) \mid x, y, z \in \mathbb{R}\}$ be

subspaces of $\mathbb{R}^4$. Find dim $(W_1 + W_2)$.

3 (a) Answer the following : **4**

(1) Define : Linear Transformation.

(2) Let V be a vector space. Prove that $T : V \to V$ defined as $T(v) = v$ is a linear transformation.

(3) Let $T : U \to V$ be a linear transformation and let θ and θ' be zero vectors of U and V respectively. Prove that $T(\theta) = \theta'$.

(4) Define : Range of a linear transformation.

(b) Attempt any **one** : **2**

(1) Let $T : U \to V$ be a linear transformation and N_T denote kernel of T. Prove that N_T is a subspace of U.

(2) Let $T : U \to V$ and $S : V \to W$ be linear transformation. Prove that the composition $S.T : U \to W$ is a linear transformation.

(c) Attempt any **one** : **3**

(1) Prove that a linear transformation $T : U \to V$ is one-one if and only if $N_T = \{\theta\}$, where N_T denotes kernel of T and θ denote zero vector of U.

(2) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y + z, y - z, z)$ be a linear transformation. Prove that T is nonsingular. Also find T^{-1}.

(d) Attempt any **one** : **5**

(1) Let $L(U, V)$ denote the set of all linear transformations from a vector space U to a vector space V. Prove that $L(U, V)$ is a vector space over $I\!R$ under usual operations.

(2) State and prove rank-nullity theorem for linear transformation.

4 (a) Answer the following : **4**

(1) Define : Linear function. Give a suitable example of it.

(2) Define : Eigenvalue of a linear transformation.

(3) If $\dim U = m$ and $\dim V = n$ then what is $\dim(L(U, V))$?

(4) If λ is an eigenvalue of a linear transformation T and B is a basis of T then $[(T - \lambda I_n) : B]$ is singular.
(True / False)

(b) Attempt any **one** : **2**

(1) Define : Matrix associated with a linear transformation.

(2) Let $T : V \to V$ be a linear transformation and $\dim(V) = n.$ If $\lambda_1, \lambda_{2, \ldots\ldots,} \lambda_n$ are distinct eigenvalues of T and $B = \{v_1, v_2, \ldots\ldots, v_n\}$ is corresponding T-eigen basis of V, then prove that $[T : B] = diag(\lambda_1, \lambda_2, \ldots\ldots, \lambda_n)$, where v_i is the eigenvector corresponding to $\lambda_i, 1 \le i \le n.$

(c) Attempt any **one** : **3**

(1) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined as $T(x, y, z) = (x + y, y + z)$. Find $[T : B_1, B_2]$, where $B_1 = \{(1, 1, 1), (1, 0, 0), (1, 1, 0)\}$ and $B_2 = \{e_1, e_2\}$ are basis of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.

(2) Find eigenvalues of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x, y) = (3y, 2x - y)$.

(d) Attempt any **one** : **5**

(1) Let $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear transformation defined as $T(p(x)) = \int_0^x p(x)\,dx$.

Let $B_1 = \{1, x, x^2\}$ and $B_2 = \{1, x, x^2, x^3\}$ be the basis of $P_2(\mathbb{R})$ and $P_3(\mathbb{R})$ respectively. Find $[T : B_1, B_2]$

(2) Find eigenvalues and eigenvectors of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x, y, z) = (x + y + z,\ x + y + z,\ x + y + z)$

5 (a) Answer the following : **4**

(1) Write the formula to find radius of curvature of the curve given by $r = f(\theta)$.

(2) Define : Point of inflexion of a curve.

(3) Show that the curve given by $y = \log x$ is concave downwards everywhere.

(4) Define : Multiple point of a given curve.

(b) Attempt any **one** : **2**

(1) Using Newton's method, find the radius of curvature at origin of the curve

$$x^4 + y^4 + x^3 - y^3 + x^2 - y^2 + y = 0.$$

(2) Find asymptotes of the curve $\left(x^2 + y^2\right)x - ay^2 = 0$ parallel to coordinate axes.

(c) Attempt any **one** : **3**

(1) Find the interval of values of x for which the curve $y = \left(x^2 + 4x + 5\right)e^{-x}$ is concave upwards or concave downwards. Also find the points of inflexion.

(2) Find oblique asymptote of the curve

$$y = \frac{x^2 + 2x - 1}{x}.$$

(d) Attempt any **one** : **5**

(1) Derive the formula to find radius of curvature of the curve given by $y = f(x)$.

(2) Discuss double points of the curve

$x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$